MAHARAJA'S COLLEGE, ERNAKULAM

(A Govt. Autonomous College)

BOARD OF STUDIES IN MATHEMATICS (PG)

CURRICULUM

FOR

M. Sc. MATHEMATICS PROGRAMME

UNDER

CHOICE BASED CREDIT SYSTEM (CBCS-PG)

(Effective from 2016 admission onwards)

PREFACE

The present time is experiencing unprecedented progress in the field of Science and technology in which mathematics is playing a vital role; and so the curriculum and syllabi of any academic programme has to be systematically subjected to thorough revision so as to make them more relevant and significant.

Maharaja's college, Ernakulam is a unique institution of higher learning in the state. Its hoary tradition and consistent achievement in various fields of human activity envelop it with a halo of an outstanding temple of knowledge.

The college was elevated to the status of autonomous College by the Government of Kerala and UGC in the year 2014. This is the only government college in Kerala which has been granted autonomy.

The College is also committed to prepare a comprehensive plan of action for Credit and semester system in Post Graduate programmes. Various workshops with the participation of the teachers from affiliated colleges and invited experts from other Universities were conducted at our institution. The syllabus and curriculum we present here is the follow-up of such workshops.

We gratefully acknowledge the assistance and guidance received from the academic and governing council of our college and all those who have contributed in different ways in this venture.

It is recommended that the content of this syllabus be reviewed and adapted in the light of the consultative process and based on its application in future curriculum revision initiatives. The syllabus and curriculum also be revised periodically.

I hope this restructured syllabus and curriculum would enrich the students.

Prof. T M Safiya
Chairman Board of Studies (PG)

MASTER DEGREE PROGRAMME IN MATHEMATICS
BOARD OF STUDIES MEMBERS
Mathematics (PG)
MAHARAJA'S COLLEGE, ERNAKULAM

(A Govt. Autonomous College)

Sl. No.	Category	Name	Designation
1	Internal	T. M. Safiya(Chair person)	Associate Professor
2	Internal	Dr.P.V.Mathai	Associate Professor
3	Internal	Dr. Bloomy Joseph	Assistant Professor
4	Internal	Jaya S.	Assistant Professor
5	Internal	Jaya Augustine	Assistant Professor
6	Internal	Thasneem T.R.	Assistant Professor
7	Internal	Murali T.K.	Assistant Professor
8	External experts	Dr. E. Krishnan	Rtd.Professor, University college, TVM
9	External experts	Dr.M.N.Narayanan Namboothiri	Rtd. Professor, CUSAT
10	Expert from Industry	P. Padmanabhan	Rtd.Scientist(ISRO)
11	Alumni Member	Dr. Mary Metilda	Rtd.Principal, Maharaja's college
12	University Nominee	Dr. Paul Isaac	Associate Professor, Bharata Mata College, Thrikkakara

MAHARAJA'S COLLEGE, ERNAKULAM
(A GOVERNMENT AUTONOMOUS COLLEGE)
REGULATIONS FOR POST GRADUATE PROGRAMMES
UNDER CHOICE BASED CREDIT SYSTEM
(2016 Admission onwards)

1. SHORT TITLE
    1.1. These Regulations shall be called as Post Graduate Regulations (2016) of Maharaja's College (Autonomous) under the Choice Based Credit System.
    1.2. These Regulations shall come into force from the Academic Year 2016-2017 onwards
2. SCOPE
    2.1. The regulation provided herein shall apply to all regular post-graduate programmes, MA/MSc / MCom, conducted by Maharaja's College (Autonomous) with effect from the academic year 2016-2017 admission onwards.
3. DEFINITIONS
    3.1. 'Programme' means the entire course of study and Examinations.
    3.2. 'Duration of Programme' means the period of time required for the conduct of the programme. The duration of post-graduate programme shall be of 4 semesters.
    3.3. 'Semester' means a term consisting of a minimum of 90 working days, inclusive of examination, distributed over a minimum of 18 weeks of 5 working days each.
    3.4. 'Course' means a segment of subject matter to be covered in a semester. Each Course is to be designed variously under lectures / tutorials / laboratory or fieldwork / seminar / project / practical training / assignments/evaluation etc., to meet effective teaching and learning needs.
    3.5. 'Credit' (Cr) of a course is a measure of the weekly unit of work assigned for that course in a semester.
    3.6. 'Course Credit' One credit of the course is defined as a minimum of one hour lecture /minimum of 2 hours lab/field work per week for 18 weeks in a Semester. The course will be considered as completed only by conducting the end semester examination. The total minimum credits required for completing a PG programme is 80.
    3.7. 'Programme Core course' Programme Core course means a course that the student admitted to a particular programme must successfully complete to receive the Degree and which cannot be substituted by any other course.
    3.8. 'Programme Elective course' Programme Elective course means a course, which can be substituted, by equivalent course from the same subject and a minimum number of courses is required to complete the programme.
    3.9. 'Programme Project' Programme Project means a regular project work with stated credits on which the student undergo a project under the supervision of a teacher in the parent department / any appropriate research center in order to submit a dissertation on the project work as specified.

3.10. ‘Seminar‘ seminar means a lecture expected to train the student in self-study, collection of relevant matter from the books and internet resources, editing, document writing, typing and presentation.

3.11. ‘Evaluation’ means every student shall be evaluated by 20% in-semester assessment and 80% end- semester assessment.

3.12. ‘Repeat course’ is a course that is repeated by a student for having failed in that course in an earlier registration.

3.13. ‘Audit Course’ is a course for which no credits are awarded.

3.14. ‘Parent Department’ means the Department which offers a particular post graduate programme.

3.15. ‘Department Council’ means the body of all teachers of a Department in a College.

3.16. ‘Faculty Advisor’ is a teacher nominated by a Department Council to coordinate the continuous evaluation and other academic activities undertaken in the Department.

3.17. ‘Letter Grade’ in a course means a letter symbol (S,A,B,C,D, etc.) which indicates the broad level of performance of a student in a course.

3.18. Each letter grade is assigned a ‘Grade point’ (GP) which is an integer indicating the numerical equivalent of the broad level of performance of a student in a course.

3.19. Credit Point (CP) of a course is the value obtained by multiplying the grade point (GP) by the Credit (Cr) of the course CP=GP x Cr.

3.20. Extra Credits are additional credits awarded to a student over and above the minimum credits required for a programme for achievements in co-curricular activities carried out outside the regular class hours as directed by the Mahatma Gandhi University

3.21. Cumulative Grade Point Average (CGPA) is the value obtained by dividing the sum of credit points in all the courses taken by the student for the entire programme by the total number of credits and shall be rounded off to two decimal places.

3.22. Grace Marks means marks awarded to course/s, as per the UO’s issued from time to time, in recognition of meritorious achievements in NSS/Sports/Arts and cultural activities.

4. PROGRAMME STRUCTURE

4.1. Students shall be admitted into post graduate programme under the various faculties.

4.2. The programme shall include two types of courses, Program Core (C) courses and Program Elective (E) Courses. There shall be a Program Project (P) with dissertation to be undertaken by all students. The Programme will also include assignments, seminars / practical, viva (V) etc., if they are specified in the Curriculum.

4.3. There shall be various groups of Programme Elective courses for a programme such as Group A, Group B etc. for the choice of students subject to the availability of facility and infrastructure in the institution and the selected group shall be the subject of specialization of the programme.

4.4. **Project work**

4.4.1. Project work shall be completed by working outside the regular teaching hours.

4.4.2. Project work shall be carried out under the supervision of a teacher in the concerned department.

4.4.3. A candidate may, however, in certain cases be permitted to work on the project in an Industrial / Research Organization on the recommendation of the Supervisor.

4.4.4. There should be an in-semester assessment and end-semester assessment for the project work.

4.4.5. The end-semester evaluation of the Project work is followed by presentation of work including dissertation and Viva-Voce.

4.5. Seminar Lectures

4.5.1 Every PG student shall deliver one seminar lecture as an internal component for every course. The seminar lecture is expected to train the student in self-study, collection of relevant matter from the books and Internet resources, editing, document writing, typing and presentation.

4.6 Test Papers

4.6.1 Every student shall undergo at least two class tests as an internal component for every course.

4.7 Assignments

4.7.1 Every student shall submit one assignment as an internal component for every course.

4.8 Attendance

4.8.1 The attendance of students for each course shall be another component of in-semester assessment.

4.8.2 The minimum requirement of aggregate attendance during a semester for appearing the end semester examination shall be 75%.

4.8.3 Condonation of shortage of attendance to a maximum of 10 days in a semester subject to a maximum of two times during the whole period of post graduate programme.

4.8.4 If a student represents his/her institution, University, State or Nation in Sports, NSS or Cultural or any other officially sponsored activities such as college union / university union activities, he/she shall be eligible to claim the attendance for the actual number of days participated subject to a maximum of 10 days in a Semester based on the specific recommendations of the Head of the Department and Principal of the College.

4.8.5 A student who does not satisfy the requirements of attendance shall not be permitted to take the end-semester examinations.

4.8.6 Those students who are not eligible even with condonation of shortage of attendance shall repeat the course along with the next batch.

4.9 Maximum Credit

4.9.1 No course shall have more than 4 credits.

4.10 Viva-Voce

4.10.1 Comprehensive Viva-voce shall be conducted at the end semester of the programme. Comprehensive Viva-Voce covers questions from all courses in the programme.

4.11 Alpha numeric code

4.11.1 Each course shall have an alpha numeric code number which includes abbreviation of the subject in three letters, the semester number, the code of the course and the serial number of the course ('C' for Program Core course, 'E' for Program Elective course, 'P' for Practicals, 'D' for Project/ Dissertation and 'V' for comprehensive Viva).

5. REGISTRATION

5.1. A student shall be permitted to register for the programme at the time of admission.

5.2. A student who registered for the course shall complete the course within a period of 8 semesters from the date of commencement of the programme.

6. ADMISSION

6.1. Candidates for admission to the first semester of the PG programme shall be required to have passed an appropriate Degree Examination of any recognized University or authority accepted by the Academic council of the Maharaja's College.

6.2. The candidate has to register all the courses prescribed for the particular semester.

6.3. Cancellation of registration is applicable only when the request is made within two weeks from the time of admission.

6.4. Students admitted under this programme are governed by the Regulations in force.

7. PROMOTION

7.1. A student who registers for the end semester examination shall be promoted to the next semester.

8. EXAMINATION

8.1. There shall be end semester examination at the end of each semester.

8.2. The answers must be written in English except for those coming under Faculty of Languages.

8.3. Practical examinations shall be conducted by the college at the end of even semesters only.

8.4. Project evaluation and Viva -Voce shall be conducted at the end of the programme only.

8.5. Practical examination, Project evaluation and Viva-Voce shall be conducted by two external examiners.

9. **END-SEMESTER EXAMINATION**

9.1. The examinations shall normally at the end of each semester. There shall be one end-semester examination of 3 hours duration in each lecture based course and practical course.

9.2. A question paper may contain short answer type/annotation, short essay type questions/problems and long essay type questions.

10. EVALUATION AND GRADING

10.1. Evaluation

10.1.1 The evaluation scheme for each course shall contain two parts; (a) in-semester evaluation and (b) end-semester evaluation. 20 marks shall be given to in-semester evaluation and the remaining 80 marks to end-semester evaluation. Both in-semester and end semester evaluation shall be carried out by using in mark system. Both internal and external marks are to be mathematically rounded to the nearest integer.

10.1.2 **Internal evaluation**

10.1.2.1 The internal evaluation shall be based on predetermined transparent system involving periodic written tests, assignments, seminars and attendance in respect of theory courses and based on written tests, lab skill/records/viva and attendance in respect of practical courses. The marks assigned to various components for in-semester evaluation is as follows.

Components of In-semester Evaluation (For theory)

Components	Component Marks
Assignment	4
Seminar	4
Two Test papers*	8
Attendance	4
Total	20

*Marks of Test Papers shall be the average

Components of In-semester Evaluation (For Practical)

Components	Component Marks
Attendance	4
Laboratory Involvement	4
Written/Lab Test	4
Record*	4
Viva	4
Total	20

*Marks awarded to Record should be related to number of experiments recorded

Components of In-semester Evaluation (For Project)

Components	**Marks**
Topic/Area selected	2
Experimentation/Data collection	4
Punctuality	2
Compilation	4
Content	4
Presentation	4
Total	**20**

a)Evaluation of **Attendance**

% of attendance	Mark
95 and above	4
85 to 94	3
80 to 84	2
75 to 79	1
< 75	0

(Decimals are to be rounded to the next higher whole number)

b) Evaluation of Assignment

Components	Marks
Punctuality	1
Content	1
Conclusion	1
Reference/Review	1
Total	**4**

c) Evaluation of Seminar

Components	Marks
Content	1
Presentation	2
Reference/Review	1
Total	**4**

10.1.2.2 To ensure transparency of the evaluation process, the in-semester marks awarded to the students in each course in a semester shall be published on the notice board at least one week before the commencement of external examination. There shall not be any chance for improvement for in semester marks.

10.1.2.3 The course teacher and the faculty advisor shall maintain the academic record of each student registered for the course and a copy should be kept in the college for at least one year for verification.

10.1.3 End-Semester Evaluation:

10.1.3.1 The end-semester evaluation in theory courses is to be conducted by the college with question papers set by external experts. The answers must be written in English except those for the Faculty of Languages. The evaluation of the answer scripts shall be done by examiners based on a well-defined scheme of valuation. The end-semester evaluation shall be done immediately after the examination preferably through Centralized Valuation.

10.1.3.2 Photocopies of the answer scripts of the external examination shall be made available to the students for scrutiny on request and revaluation/scrutiny of answer scripts shall be done as per the request of the candidate by paying fees.

10.1.3.3 The question paper should be strictly on the basis of model question paper set by BOS and there shall be a combined meeting of the question paper setters for scrutiny and finalization of question paper. Each set of question should be accompanied by its answer scheme for valuation.

10.1.3.4 **Pattern of Questions**

10.1.3.4.1 The question setter shall ensure that questions to course should satisfy weightage to objectives and weightage to difficulty levels.

Weightage to Objectives

Objectives	%
Understanding	25
Critical Evaluation	50
Application	25

Weightage to difficulty levels

Level of difficulty	%
Easy	20
Average	60
Difficult	20

10.1.3.4.2 Question paper setters shall also submit a detailed scheme of evaluation along with the question paper. A question paper shall be a judicious mix of objective type, short answer type, short essay type /problem solving type and long essay type questions.

Pattern of questions for end semester examination

	Total no. of questions	Number of questions to be answered	Marks of each question	Total marks
	12	10	2	20
	10	6	5	30
	4	2	15	30
TOTAL	**26**	**18**	x	**80**

10.2 Grades for Courses

For all courses (theory & practical), grades are given on a 10-point scale based on the total percentage of marks ***(ISA+ESA)*** as given below

Percentage of Marks	Grade	Grade Point (GP)
95 and above	S Outstanding	10
85 to below 95	A^+ Excellent	9
75 to below 85	A Very Good	8
65 to below 75	A- Good	7
55 to below 65	B^+ Above Average	6
50 to below 55	B Average	5
40 to below 50	C Pass	4
Below 40	F Fail	0
	Ab Absent	0

11. CREDIT POINT AND CREDIT POINT AVERAGE

Credit Point (CP) of a course is calculated using the formula

CP = C × GP, where C = Credit; GP = Grade point

Semester Grade Point Average (SGPA) of a Semester is calculated using the formula

SGPA= TCP/TC, where TCP = Total Credit Point of that Semester

TC = Total Credit of that Semester

Cumulative Grade Point Average (CGPA) of a Programme is calculated using the formula

CGPA = ∑(TCP ×TC)÷∑TC

CGPA shall be rounded off to two decimal places

12. Grades for the different semesters and overall programme are given based on the corresponding CPA as shown below:

GPA	Grade
Equal to 9.5 and above	***S Outstanding***
Equal to 8.5 and below 9.5	***A+ Excellent***
Equal to 7.5 and below 8.5	***A Very Good***
Equal to 6.5 and below 7.5	***A- Good***
Equal to 5.5 and below 6.5	***B+ Above Average***
Equal to 5.0 and below 5.5	***B Average***
Equal to 4.0 and below 5.0	***C Pass***
Below 4.0	***F Failure***

12.1.A separate minimum of 40% marks each for in-semester and end semester (for both theory and practical) and aggregate minimum of 40% are required to pass for a course. To pass in a programme, a separate minimum of Grade **C** is required for all the individual courses. If a candidate secures **F** Grade for any one of the courses offered in a Semester/Programme only **F** grade will be awarded for that Semester/Programme until he/she improves this to **C** grade or above within the permitted period. Candidates who secures **C (CGPA)** grade and above shall be eligible for higher studies.

12.2. A candidate who has not secured minimum marks/credits in internal examinations can re-do the same registering along with the end-semester examination for the same semester, subsequently.

12.3. A student who fails to secure a minimum marks/grade for a pass in a course will be permitted to write the examination along with the next batch.

12.4. There will be no supplementary examinations. A candidate will be permitted to improve the marks/CGPA of a programme within a continuous period of four semesters immediately following the completion of the programme. If a candidate opts for the betterment of a programme, he/she has to appear for the entire semester. The consolidation of marks/grade/grade points after the betterment examination is limited to one time

13. AWARD OF DEGREE

The successful completion of all the courses with CGPA of 'C' (40%) shall be the minimum requirement for the award of the degree.

13. GRIEVANCES REDRESS COMMITTEE

The College shall form a Grievance Redress Committee in each Department comprising of course teacher and one senior teacher as members and the Head of the Department as Chairman. The Committee shall address all grievances relating to the in-semester assessment grades of the students. There shall be a college level Grievance Redress Committee comprising of Faculty advisor, two senior teachers and the Principal as Chairman.

Department of Mathematics

Mathematics **Total Credits: 80**

Curriculum

e Code	Course	Credit	Marks			Weekly Contact Hours		Course Code	Course	Credit	Marks	
			Int.	Ext.	Total						Int.	Ext.
ATC01	Linear Algebra	4	20	80	100	5	**Semester II**	PG2MATC06	Abstract Algebra	4	20	80
ATC02	Metric space	4	20	80	100	5		PG2MATC07	Topology I	4	20	80
ATC03	Real Analysis	4	20	80	100	5		PG2MATC08	Advanced Complex Analysis	4	20	80
ATC04	Graph Theory	4	20	80	100	5		PG2MATC09	Ordinary Differential Equation	4	20	80
ATC05	Complex Analysis	4	20	80	100	5		PG2MATC10	Measure Theory and Integration	4	20	80
	TOTAL	**20**			**500**	**25**			**TOTAL**	**20**		
ATC11	Analytic Number Theory	4	20	80	100	5	**Semester IV**	PG4MATC16	Spectral Theory	3	20	80
ATC12	Functional Analysis	4	20	80	100	5		PG4MATE01	Number Theory and Cryptography	3	20	80
ATC13	Topology II	4	20	80	100	5		PG4MATE02	Multivariate Calculus, Integral Transforms and Manifolds	3	20	80
ATC14	Partial Differential Equations and Integral Equation	4	20	80	100	5		PG4MATE03	Differential Geometry	3	20	80
ATC15	Optimization Techniques	4	20	80	100	5		PG4MATE04	Theory of Wavelets	3	20	80
								PG4MATD01	Project	3	20	80
								PG4MATV01	Viva	**2**		
	TOTAL	**20**			**500**	**25**			**TOTAL**	**20**		

Semester - 1

PG1MATC01

(5 hrs/week) **LINEAR ALGEBRA** **(80 marks)**

Text Book: Kenneth Hoffman / Ray Kunze (Second Edition), *Linear Algebra*, Prentice-Hall of India Pvt. Ltd., New Delhi, 1992.

Module 1	Vector spaces, Subspaces, basis and dimension (Proof of theorems excluded) Co-ordinates, summary of row-equivalence (Chapter 2,2.1 - 2.5 of the text)	(15 hours.)
Module 2	Linear transformations, the algebra of linear transformations, isomorphism, representation of transformations by matrices, linear functionals, double dual, transpose of a linear transformation (Chapter 3 - 3.1, 3.2, 3.3, 3.4, 3.5, 3.6 & 3.7 of the text)	(30 hours.)
Module 3	Determinants: Commutative Rings, Determinant functions, Permutation and uniqueness of determinants, Additional properties of determinants. (Chapter 5 - 5.1, 5.2, 5.3 & 5.4 of the text)	(18 hours.)
Module 4	Introduction to elementary canonical forms, characteristic values, annihilatory polynomials, invariant subspaces, simultaneous triangulations, simultaneous diagonalisation, direct sum decompositions, invariant direct sums (Chapter 6 - 6.1, 6.2, 6.3, 6.4, 6.5 & 6.6 of the text)	(27 hours.)

Question paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	3	2	1
Module II	3	3	1
Module III	3	2	1
Module IV	3	3	1
Total	**12**	**10**	**4**

References:

1.Klaus Jonich. Linear Algebra, Springer Verlag.

2.Paul R. Halmos, Linear Algebra Problem Book, The Mathematical Association of America.

3 S. Kumaresan, Linear Algebra A Geometrical Approach, Prentice Hall of India, 2000.

PG1MATC02

(5 hrs/week) **METRIC SPACES** **(80 marks)**

Text Book: "Topology and Modern Analysis" by George F . Simmons, McGraw-Hill Book Company, International edition, 1963.

Module 1	A quick revision of the ideas given in the first chapter of the Prescribed text book. Metric spaces: The definition and some examples, Open sets, Closed sets, Convergence, completeness, Baire's theorem. Chapter 2 : sections 9, 10, 11 and 12.	(20 hrs)
Module 2	Continuous mappings, Spaces of continuous functions, Euclidean and unitary spaces. Chapter 2 : sections 13, 14 and 15.	(25 hrs)
Module 3	Topological spaces: The definition and some examples, Elementary concepts, Open bases and open Sub bases, Weak topologies, The function algebras C (X , R) and C (X , C) . Chapter 3 : sections 16, 17, 18, 19 and 20. Proof of Theorem B in section 20 is excluded.	(20 hrs)
Module 4	Compactness: Compact spaces, Product of spaces, Tychonoff 's theorem and locally compact spaces, Compactness for metric spaces. Chapter 4 : sections 21, 22, 23 and 24.	(25 hrs)

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	3	1
Module III	3	2	1
Module IV	3	3	1
Total	**12**	**10**	**4**

References :

1. Topology - A first course by James R. Munkres, Prentice Hall India.
2. General Topology by Stephan Willard, Addison-Wesley, 1970.
3. Introduction to general topology by K. D. Joshi, Wiley Eastern Limited,1983.

REAL ANALYSIS PG1MAT03

(5 hrs/week) (80marks)

Text 1: Tom Apostol, Mathematical Analysis (second edition), Narosa Publishing House.

Text 2: Walter Rudin, Principles of Mathematical Analysis (Third edition), International Student Edition.

Pre-requisites: A quick review on continuity, uniform continuity, convergence of sequence and series.

(No question shall be asked from this section.)

Module 1	**Functions of bounded variation and rectifiable curves** Introduction, properties of monotonic functions, functions of bounded variation, total variation, additive property of total variation, total variation on (a, x) as a functions of x, functions of bounded variation expressed as the difference of increasing functions, continuous functions of bounded variation, curves and paths, rectifiable path and arc length, additive and continuity properties of arc length, equivalence of paths, change of parameter. (Chapter 6, Section: 6.1- 6.12. of Text 1)	(20 hrs.)
Module 2	**The Riemann-Stielljes Integral** Definition and existence of the integral, properties of the integral, integration and differentiation, integration of vector valued functions. (Chapter 6 - Section 6.1 to 6.25 of Text 2)	(25 hrs.)
Module 3	**Sequence and Series of Functions** Discussion of main problem, uniform convergence, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, the Stone-Weierstrass theorem (without proof). (Chapter 7 Section. 7.7 to 7.18 of Text 2)	(25 hrs)
Module 4	**Some Special Functions** Power series, the exponential and logarithmic functions, the trigonometric functions, the algebraic completeness of complex field, Fourier series. (Chapter 8 - Section 8.1 to 8.16 of Text 2)	(20 hrs)

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	3	1
Module III	3	3	1
Module IV	3	2	1
Total	12	10	4

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References:-

1. Royden H.L, Real Analysis, 2^{nd} edition, Macmillan, New York.
2. Bartle R.G, The Elements of Real Analysis, John Wiley and Sons.
3. S.C. Malik, Savitha Arora, Mathematical Analysis, New Age International Ltd.
4. Edwin Hewitt, Karl Stromberg, Real and Abstract Analysis, Springer International, 1978.

PG1MATC04

(5 hrs/week) GRAPH THEORY (80marks)

Text : R.Balakrishnan and K. Ranganathan, A Text book of Graph Theory, Springer

Module 1	**Basic results and directed graphs** Basic concepts. sub graphs. degrees of vertices. Paths and connectedness automorphism of a simple graph, line graphs, basic concepts and tournaments. **Connectivity** Vertex cuts and edge cuts. connectivity and edge connectivity, blocks. (Chapter 1 Sections 1.1 to 1.5 and 1.6 (Up to 1.6.3) Chapter 2 Sections 2.1 and 2.2 Chapter 3 Sections 3.1 to 3.3 of the text)	(20 hrs.)
Module 2	**Trees:** Definition, characterization and simple properties, centres and cenroids, counting the number of spanning trees, Cayley's formula, applications (Chapter 4 Sections 4.1 to 4.4 Chapter 10 Sections 10.1 to 10.4 of the text)	(20 hrs.)
Module 3	Independent Sets, Eulerian Graphs; Hamiltonian Graphs and Vertex Colouring, Vertex independent sets and vertex coverings. edge independent sets, Eulerian graphs, Hamiltonian graphs, vertex colourings, critical graphs, triangle free graphs. (Chapter 5 Sections 5.1 and 5.2 Chapter 6 Sections 6.1 and 6.2 Chapter 7 Sections 7.1 to 7.3 of the text)	(25 hrs)
Module 4	Edge coloring and planarity- Edge coloring of graphs, planar and non-planar graphs, Euler formula and its consequences, K5 and K3,3 are non-planar graphs, dual of a plane graph. the four color theorem and Heawood five color theorem. (Chapter 7 Section 7.4 Chapter 8 Sections 8.1 to 8.5 of the text)	(25 hrs)

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	3	1
Total	**12**	**10**	**4**

References:

1. John Clark and Derek Allan Holton, A First Look at Graph Theory, Allied Publishers.
2. Douglas B West, Introduction to Graph Theory, Prentice Hall of India
3. F.Harary, Graph Theory, Addison-Wesley, 1969.

PG1MATC05

(5hrs/week) COMPLEX ANALYSIS (80marks)

Text: John B. Conway , Functions of One Complex Varible, Second Edition

Module 1	**Analytic functions** Power series , Analytic functions , Analytic Functions as mappings, Mobius transformations (Chapter 3 of the text)	(20 hrs.)
Module 2	**Complex Integration** Power series representation of analytic functions, Zeros of an analytic function , The index of a closed curve , Cauchy's Theorem and Integral Formula, The Homotopic version of Cauchy's Theorem and simple connectivity (Chapter 4 sections 4.2- 4.6 of the text)	(20 hrs.)
Module 3	**Zeros and Singularities** Counting zeros, the open mapping Theorem, Gorsat's Theorem, Classification of singularities, Residues, The Argument Principle. (Chapter 4 sections 7 & 8 of the text)	(25 hrs)
Module 4	**The Maximum Modulus Theorem** The Maximum Principle , Schwarz's Lemma, Convex Functions and Hadamard's Three circles Theorem (Chapter 6 sections 1 -3)	(25 hrs)

Question paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	3	2	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	3	1
Total	**12**	**10**	**4**

References:

1. Chaudhary. B, The elements of Complex Analysis, Wiley Eastern.
2. Cartan. H (1973), Elementary theory of Analytic functions of one or several variable, Addison Wesley.
3. Lang. S, Complex Analysis, Springer.
4. H.A. Priestly, Introduction to Complex Analysis, Clarendon press, Oxford, 1990.

Semester 2

PG2MATC06

(5Hrs/week) ABSTRACT ALGEBRA (80marks)

Text Book: John B. Fraleigh, A First Course in Abstract Algebra, 7th edition, Pearson Education.

Module 1:	Direct products and finitely generated Abelian groups, fundamental theorem (without proof), Applications Rings of polynomials, factorisation of polynomials over a field. (Part II – Section 11) & (Part IV – Sections 22 & 23)	(25 hours)
Module 2:	Introduction to extension fields, algebraic extensions, Geometric constructions. Finite fields. (Part VI – Section 29, 31 – 31.1 to 31.18, 32, 33)	(25 hours)
Module 3:	Sylow's theorems, Applications of sylow theory Automorphism of fields, the isomorphism extension theorem (proof of the theorem excluded) (Part VII Sections 36 & 37) (Part X – Sections 48 & 49, (49.1 to 49.5)	(20 hours)
Module 4:	Splitting fields, separable extensions, Galois theory (Part X – Sections 50, 51, 53 -53.1 to 53.6)	(20 hours)

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	3	1
Module III	3	2	1
Module IV	3	2	1
Total	**12**	**10**	**4**

References:-

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd., New Delhi, 1975.

2. Hungerford, Algebra, Springer

3. M. Artin, Algebra, Prentice -Hall of India, 1991

4. N. Jacobson, Basic Algebra Vol. I, Hindustan Publishing Corporation

PG2MAT07

(5Hrs/week) TOPOLOGY-I (80 marks)

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Prescribed Text Book: "General Topology" by Stephan F . Willard, Addison-Wesley , 1970.

Module 1	A quick revision of the ideas given in the first chapter of the Prescribed text book. Topological spaces: Fundamental concepts, Bases and subbases. Chapter 2 : sections 3 , 4 and 5.	(24 hrs)
Module 2	New spaces from Old, Continuous functions, Product spaces, weak topologies . Chapter 3 : sections 6, 7 and 8.	(22 hrs)
Module 3	Quotient spaces. Convergence : Inadequacy of sequences ,Nets. Chapter 3 : section 9. Chapter 4 : sections 10 , 11.	(22 hrs)
Module 4	Filters. Separation and Countability: Separation axioms, Regularity and Complete regularity. Chapter 4: section 12. Chapter 5: sections 13, 14.	(22 hrs)

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	3	1
Module III	3	2	1
Module IV	3	2	1
Total	**12**	**10**	**4**

References :

1. Topology - A first course by James R. Munkres, Prentice Hall India.
2. Introduction to Topology andModern Analysis by George F . Simmon, McGraw-Hill, 1963.
3. Introduction to general topology by K. D. Joshi, Wiley Eastern Limited,1983.

PG2MATC08

(5Hrs/week) ADVANCED COMPLEX ANALYSIS (80marks)

Text Book: John B. Conway, Functions of One Complex Variable, Second Edition

Module 1:	Compactness and Convergence in the Space of Analytic Functions The space of continuous functions , spaces of analytic functions, The Riemann Mapping Theorem , Weirerstrass Factorization Theorem, Factorization of the sine function ,The gamma function , The Riemann zeta function (Chapter 7 Sections 1,2,4-8)	(30 hours)
Module 2:	Runge's Theorem Runge's Theorem, Simple connectedness, Mittag Leffler's Theorem, Analytic Continuation and Riemann Surfaces , Schwarz Reflection Principle, Analytic Continuation along a path, Mondromy Theorem(without proof) (Chapter 8 Sections 1-3, Chapter 9 Sections 1-3)	(25 hours)
Module 3:	Harmonic Functions Basic Properties of harmonic functions , Harmonic functions on a disk, (Chapter 10 Sections 1-2)	(20 hours)
Module 4:	Entire Functions Jensen's Formula, The genus and order of an entire function , Hadamard Factorization Theorem (without proof) (Chapter 11 Sections 1-3)	(15 hours)

Question paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	3	3	1
Module II	3	3	1
Module III	3	2	1
Module IV	3	2	1
Total	**12**	**10**	**4**

References:

1. Chaudhary. B, The elements of Complex Analysis, Wiley Eastern.
2. Cartan. H (1973), Elementary theory of Analytic functions of one or several variable, Addison Wesley.
3. Lang. S, Complex Analysis, Springer.

PG2MATC09

(5hrs/week) **Ordinary Differential Equations** **(80 marks)**

Text Book: Ordinary Differential Equations and Stability theory: by S G Deo and V Raghavendra.TMH-1980.

Module-I	**Linear Differential Equations.:** Introduction, how differential equations arise, a simple equation, separable equations, Purpose of theoretical considerations, Classification, Initial and boundary value problems, Linear dependence and Wronskian, Basic theory for linear equations, method of variation of parameters, two useful formulae, Homogeneous linear equations with constant coefficients (Chapters 1 and 2 of the text book)	**(20hrs)**
Module-II	**Power series solutions:** Introduction, second order linear equations with ordinary points, Legendre equation and Legendre polynomials, Second order equations with regular singular points, Bessel equation (All sections of chapter 3 in the text book.)	**(22 hrs)**
Module-III	**System Linear Differential Equations.:** Systems of first order equations, Existence and uniqueness theorem, Fundamental matrices, Non homogeneous linear systems, linear systems with constant co-efficient, linear systems with periodic co-efficient (All sections of chapter 4 in the text book.)	**(24 hrs)**
Module-IV	**Existence and uniqueness of solutions.:** Preliminaries, successive approximation, Picard's theorem, non uniqueness of solutions, continuation and dependence on initial conditions, existence of solutions in the large, Strum -liouville's problem, Green's functions (All sections except section 5.8 of chapter 5 and the first three sections of chapter 7 in the text book.)	**(24 hrs)**

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	3	1
Total	**12**	**10**	**4**

References:

1. Introduction to Ordinary Differential Equations: by E A Coddington. PHI-1961
2. Differential Equations With Applications and Historical Notes: by G F Simmons. McGraw Hill-1972

PG2MAT10

(5 Hrs/week) MEASURE THEORY AND INTEGRATION (80 Marks)

Text 1: H.L. Royden, Real Analysis, Third edition, Prentice Hall of India Private Limited.

Text 2: G. de Barra, Measure Theory and Integration, New Age International (P) Linnilect Publishers.

Pre-requisites: Algebras of sets, the axiom of choice and infinite direct products, open and closed sets of real numbers. (Chapter 1 - section 4, 5, Chapter 2 - section 5 of Text 1).

(No questions shall be asked from this section)

Module 1:	Lebesgue measure: introduction, outer measure, measurable sets and Lebesgue measure & non-measurable sets, measurable functions. (Chapter 3 - Sec. 1 to 5. of Text 1)	(20 hours)
Module 2:	Lebesgue integral: the Riemann integral, he Lebesgue integral of a bounded function over a set of finite measures, the integral of a non-negative function, the general Lebesgue integral, differentiation of monotone functions. (Chapter 4 - Sec. 1 – 4. and Chapter 5 - Sec. 1. of Text 1)	(25 hours)
Module 3:	Measure and integration: measure spaces, measurable functions, Integration, general convergence theorems, signed measures, the Radon-Nikodym theorem, outer measure and measurability, the extension theorem. (Chapter 11 - Sec. 1 to 6 and Chapter 12 - Sec. 1& 2 of Text 1)	(25 hours)
Module 4:	Convergence: convergence in measure, almost uniform convergence, measurability in a product space, the product measure . (Chapter 8 - Sec. 8.1 & 8.2 of Text 2)	(20 hours)

Question paper pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	3	1
Module III	3	3	1
Module IV	3	2	1
Total	**12**	**10**	**4**

References:-

1. Halmos P.R, Measure Theory, D.van Nostrand Co.
2. P.K. Jain and V.P. Gupta, Lebesgue Measure and Integration, New Age International (P) Ltd., New Delhi, 1986(Reprint 2000).
3. R.G. Bartle, The Elements of Integration, John Wiley & Sons, Inc New York, 1966.

Semester – 3 **PG3MATC11**

(5 Hrs/week) **ANALYTIC NUMBER THEORY** **(80 marks)**

Text: Tom M.Apostol- Introduction to Analytic Number Theory, Springer International, Student Edition, Narosa Publishing House

Module 1:	**Arithmetic Functions Dirichlet's Multiplication and averages of arithmetical functions.** Introduction to chapter 1 of the text, Mobius Function $\mu(n)$, The Euler Totient Function$\varphi(n)$, A relation connecting $\mu(n)$ and $\varphi(n)$, the Dirichlet's product of arithmetical functions, Dirichlet;s Inverses and Mobius inversion formula, the Mangoldt Function $\Lambda(n)$, Mutiplicative e functions and Dirichlet's multiplication, the inverse of completely multiplicative functions, the Lioville's function $\lambda(n)$, the divisor function $\sigma_{\alpha}(n)$, generalized convolutions, formal power series, the Bell series of an arithmetical function , bell series and Dirichlet;s multiplication Introduction to chapter 2 of the text, the Big oh notation, asymptotic equality of the functions, Euler's summation formula, some elementary asymptotic formulas, the average order of $d(n)$, the average order of the divisor function$\sigma_{\alpha}(n)$, the average order of $\varphi(n)$, an application of distribution of lattice points visible from the origin, average order of $\mu(n)$ and $\Lambda(n)$, the partial sum of a dirichlet's product, application to $\mu(n)$ and $\Lambda(n)$ (chapter2 sections 2.1to2.17, chapter 3 section 3.1 to 3.11 of the text)	(30 hours)
Module 2:	**Some Elemetary theorems on the distribution of prime numbers** Introduction to chapter 4, Chebyshev's functions $\psi(x)$ and $\Im(x)$, relation connecting $\Im(x)$ and $\pi(x)$, some equivalenbt forms of prime number theorem, inequalities of $\pi(n)$and $\rho(n)$, Shapiro's Tauberian theorem, applications of Shapiro's theorem, an asymptotic formula for the partial sum $\sum_{p\leq x}\frac{1}{p}$ (Chapter 4 sections: 4.1-4.8of the text)	(20 hours)
Module 3:	**Congruences** Definition and basic properties of congruences, residue classes and complete residue systems, liner congruences, reduced residue systems and Euler – Fermat theorem, Polynomial congruences modulo p , Lagrange's theorem, applications of Lagrange's theorem, simultaneous linear congruences, the Chinese reminder theorem, applications of Chinese reminder theorem, polynomial congruences with prime power moduli (Chapter 5 sections 5.1 to 5.9 of the text)	(25 hours)

Module 4:	**Primitive roots and partitions** The exponent of a number mod m. Primitive roots, Primitive roots and reduced systems, The nonexistence of Primitive roots mod 2α for α ≥ 3 , The existence of Primitive roots mod p for odd primes p, Primitive roots and quadratic residues. Partitions – Introduction, Geometric representation of partitions, Generating functions for partitions, Euler's pentagonal-number theorem. (Chapter 10 sections 10.1 to 10.5 & Chapter 14 sections 14.1 to 14.4 of the text)	(15 hours)

Question Paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	2	1
Total	12	10	4

References:

1. Hardy G.H and Wright E.M , Introduction to the Theory of numbers, Oxford, 1981
2. Leveque W.J, Topics in Number Theory, Addison Wesley, 1961.
3. J.P Serre, A Course in Arithmetic, GTM Vol. 7, Springer-Verlag, 1973

PG3MATC12

(5Hrs/ week) **FUNCTIONAL ANALYSIS** **(80 marks)**

Text Book: Erwin Kreyszig, Introductory Functional Analysis with applications, John Wiley and sons, New York

Module 1:	Vector Space, normed space. Banach space, further properties of normed spaces, finite dimensional normed spaces and subspaces, compactness and finite dimension, linear Operators, bounded and continuous linear operators. (Chapter 2 - Sections 2.1 – 2.7 of the text)	(20 hours)
Module 2:	Linear functionals, linear operators and functionals on finite dimensional spaces, normed spaces of operators. dual space, inner product space. Hilbert space, further properties of inner product space.(Chapter 2 - Section 2.8 to 2.10, chapter 3 - Sections 3.1 to 3.2 of the text)	(20 hours)
Module 3:	Orthogonal complements and direct sums, orthonormal sets and sequences, series related to orthonormal sequences and sets, total orthonormal sets and sequences. Representation of functionals on Hilbert spaces, Hilbert adjoint operators, Self adjoint, unitary and normal operators. (Chapter 3 - Sections 3.3 to 3.6, 3.8 to 3.10 of the text)	(25 hours)
Module 4:	Zorn's lemma, Hahn- Banach theorem, Hahn- Banach theorem for complex vector spaces and normed spaces, adjoint operators, reflexive spaces, category theorem (Statement only), uniform boundedness theorem (Chapter 4 – Sections 4.1 to 4.3, 4.5 to 4.7 ofthe text)	(25 hours)

Question paper pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	2	1
Module IV	3	3	1
Total	**12**	**10**	**4**

References:

1. Simmons, G.F, Introduction to Topology and Modern Analysis, McGraw –Hill, New York 1963.

2. Somasundaram. D, Functional Analysis, S.Viswanathan Pvt. Ltd, Madras, 1994

3. M. Thamban Nair, Functional Analysis, A First Course, Prentice – Hall of India Pvt. Ltd, 2008

4. Walter Rudin, Functional Analysis, TMH Edition, 1974.

PG3MATC13

(5 Hrs/week) **TOPOLOGY II** **(80 marks)**

Prescribed Text Book: "General Topology" by Stephan F . Willard, Addison-Wesley, 1970.

Module 1:	Normal spaces , Countability properties Chapter 5 : sections 15 , 16.	(25 hours)
Module 2:	Compactness, Compactification. Chapter 6: sections 17 , 18.	(20 hours)
Module 3:	Metrizable spaces, Metrization, Baire theorem. Chapter 7: sections 22, 23, 25. Proof of the Theorem 23.9 is excluded.	(25 hours)
Module 4:	Connectedness, Pathwise and Local connectedness. Chapter 8: section 26 , 27.	(20 hours)

Question paper pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	2	1
Total	**12**	**10**	**4**

References:-

1. Munkres J.R, Topology-A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
2. Stephen Willard, General Topology, Addison-Wesley.
3. Dugundji, Topology, Universal Book Stall, New Delhi.
4. George F Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, 1963.

PG3MATC14

PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

(5Hrs/week) **(80 marks)**

Text1: Partial differential equations- Amaranath. M (Narosa,1997)

Text2: Methods of applied mathematics-Hildbrand. F.B (PHI.1972,II nd Edition)

Module-I	**First Order PDE** curves and surfaces , 1.2 genesis of first order P.D.E ; classification of integrals, linear equations. pfaffian differential equations , compatible systems charpit's method , Chapter 1 Sections 1.1 to 1.7 of Text 1	(20hrs)
Module-II	**First Order PDE(continued), Second order PDE** integral surfaces through a given curve , quasi-linear equatins , non- linear first order P.D.E genesis of second order P.D.E, classification of second order P.D.E , one dimensional wave equation,vibrations of an infinite string , vibrations of a semi- infinite string vibrations of string of finite length Chapter 1 Sections 1.9 to 1.11(omit proof of 1.11.1), Chapter 2 Sections 2.1 to 2.3.3 of text 1	(25 hrs)
Module-III	**Second Order PDE(continued)** boundary value problems , maximum and minimum principles ,the Cauchy problem ,the dirichlet problem for a circle , the Neumann problem for the upper half plane , The dirchlet problem for a circle , the dirchlet exterior problem for a circle , the Neumann problem for a circle , the dirchlet problem for a rectangle , harnack's theorem , heat conduction – infinite rod case , heat conduction – finite rod case Chapter 2 Sections 2.4.1 to 2.4.10, 2.5.1, 2.5.2 up to exer-cise2.5.1 of text **1**	(25 hrs)
Module-IV	**Integral Equations** introduction , relation between differential and integral equations , The green's function , Alternative definition of the green's function , Linear equations with seperable kernels , illustrative examples ,Hilbert – Schmidt theory , iterative methods for solving equations of the second kind The Neumann series Chapter 3 up to 3.10 (omit sections 3.4 and 3.5) of text 2)	(20 hrs)

Question paper pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	3	1
Module III	3	3	1
Module IV	3	2	1
Total	**12**	**10**	**4**

References:

1. Sneddon,I:Elements of partial differential equations-McGraw-Hill (1975)
2. John F : partial differential equations-Narosa.1986
3.M.D.Raisinghania-Integral equations and boundary value problems

PG3MATC15

(5hrs/week) OPTIMIZATION TECHNIQUES (80 marks)

Text – 1 K.V. Mital and C. Mohan, Optimization Methods in Operation Research and Systems Analysis, 3rd edition.
Text -2- Ravindran, Philips and Solberg. Operations Research Principle and Practice, 2nd edition, John Wiley and Sons.

Module-I	**INTEGER PROGRAMMING:** I.L.P in two dimensional space – General I.L.P. and M.I.L.P problems – cutting planes – remarks on cutting plane methods – branch and bound method – examples – general description – the 0 – 1 variable. (Chapter 6; sections: 6.1 – 6.10 of text – 1)	(20 hours)
Module-II	**SENSITIVITY ANALYSIS;FLOW AND POTENTIALS IN NETWORKS;** Introduction- changes in b_i - changes in c_j - changes in a_{ij}- mintroduction of new variables – introduction of new constraints – deletion of variables, deletion of constraints-goal programming ,Graphs-definitions and notations –minimum path problem-spanning tree of minimum length-problem of minimum potential difference- scheduling of sequential activities- maximum flow problem-generalized problem of maximum flow.(Chapter 5&7section 5.1-5.9&7.1-7.9,7.15 textI)	(25 hours)
Module-III	**THEORY OF GAMES:** Matrix (or rectangular) games – problem of games – minimax theorem, saddle point – strategies and pay off – theorems of matrix games – graphical solution – notion of dominance – rectangular game as an L.P. problem. (Chapter 12; Sections: 12.1 – 12.9 of text – 1)	(25 hours)
Module-IV	**NON- LINEAR PROGRAMMING:** Basic concepts – Taylor's series expansion – Fibonacci Search - golden section search – Hooke and Jeeves search algorithm – gradient projection search – Lagrange multipliers – equality constraint optimization, constrained derivatives – project gradient methods with equality constraints – non-linear optimization: Kuhn-Tucker conditions – complimentary Pivot algorithms. (Chapter 8; Sections: 8.1 – 8.14 of text – 2)	(20 hours)

Question Paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	3	1
Module III	3	3	1
Module IV	3	2	1
Total	**12**	**10**	**4**

Reference:-

1. S.S. Rao, Optimization Theory and Applications, 2nd edition, New Age International Pvt.
2. J.K. Sharma, Operations Research: Theory and Applications, Third edition, Macmillan India Ltd.
3. Hamdy A. Thaha, Operations Research – An Introduction, 6th edition, Prentice Hall of India Pvt. Ltd.

Semester – 4

PG4MATC16

(5 Hrs/week) **SPECTRAL THEORY** **(80 marks)**

Text Book: Erwin Kreyszig, Introductory Functional Analysis with applications, John Wiley and sons, New York

Module 1:	Strong and weak convergence, convergence of sequence of operators and functionals, open mapping theorem, closed linear operators, closed graph theorem, Banach fixed point theorem (Chapter 4 - Sections 4.8, 4.9, 4.12 & 4.13 - Chapter 5 – Section 5.1 of the text)	(25 hours)
Module 2:	Spectral theory in finite dimensional normed space, basic concepts, spectral properties of bounded linear operators, further properties of resolvant and spectrum, use of complex analysis in spectral theory, Banach algebras, further properties of Banach algebras. (Chapter 7 - Sections 71. to 7.7 of the text).	(25 hours)
Module 3:	Compact linear operators on normed spaces, further properties of compact linear operators, spectral properties of compact linear operators on normed spaces, further spectral properties of compact linear operators (Chapter 8 - Sections 8.1 to 8.4 of the text)	(20 hours)
Module 4:	Spectral properties of bounded self adjoint linear operators, further spectral properties of bounded self adjoint linear operators, positive operators, projection operators, further properties of projections (Chapter 9 - Sections 9.1, 9.2, 9.3, 9.5, 9.6 of the text)	(20 hours)

Question Paper Pattern

	Part A Short questions	Part B Short essays	Part C Long essays
Module I	3	3	1
Module II	3	3	1
Module III	3	2	1
Module IV	3	2	1
Total	**12**	**10**	**4**

References:-

1. Simmons, G.F, Introduction to Topology and Modern Analysis, McGraw –Hill, New York 1963.
2. Siddiqi, A.H, Functional Analysis with Applications, Tata McGraw –Hill, New Delhi 1989
3. Somasundaram. D, Functional Analysis, S.Viswanathan Pvt Ltd, Madras, 1994
4. Vasistha, A.R and Sharma I.N, Functional analysis, Krishnan Prakasan Media (P) Ltd, Meerut: 1995-96

ELECTIVE COURSES

PG4MATE01

(5 Hrs/week) NUMBER THEORY AND CRYPTOGRAPHY (80 marks)

Text Book: Neal Koblitz, A **Course in Number Theory and Cryptography, 2nd edition, Springer Verlag.**

Module 1:	**Some topics in Elementary Number Theory:-**Time estimates for doing arithmetic, divisibility and the Euclidean algorithm, congruences, some applications to factoring. (Chapter – I Sections 1, 2, 3 & 4 of the text)	(28 hours)
Module 2:	**Finite Fields and Quadratic Residues:-** Finite fields, quadratic residues and reciprocity (Chapter – II Sections 1 & 2 of the text)	(14 hours)
Module 3:	Public Key: - The idea of public key cryptography, RSA, Discrete log. (Chapter – IV Sections 1, 2 & 3 of the text)	(25 hours)
Module 4:	Primality and Factoring: -Pseudo primes, The rho method, Fermat factorization and factor bases, the quadratic sieve method. (chapter – V Sections 1, 2, 3 & 5 of the text)	(23 hours)

Question paper pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	2	1
Module IV	3	3	1
Total	**12**	**10**	**4**

Reference Books:

1. Niven, H.S. Zuckerman and H.L. Montgomery, An introduction to the theory of numbers, John Wiley, 5th Edition.
2. Ireland and Rosen, A Classical Introduction to Modern Number Theory. Springer, 2nd edition, 1990.
3. David Burton, Elementary Number Theory and its applications, Mc Graw-Hill Education (India) Pvt. Ltd, 2006.
4. Alfred J. Menezes, Paul C. van Oorschot and Scott A. Vanstone, Handbook of Applied Cryptography, CRC Press, 1996

PG4MATE02

MULTIVARIATE CALCULUS INTEGRAL TRANSFORMS AND MANIFOLDS

(5 Hrs/week) (80 marks)

Text 1: Tom APOSTOL, Mathematical Analysis, Second edition, Narosa Publishing House.

Text 2: Michael Spivak, Calculus on Manifolds

Module 1:	The Weirstrass theorem, other forms of Fourier series, the Fourier integral theorem, the exponential form of the Fourier integral theorem, integral transforms and convolutions, the convolution theorem for Fourier transforms. (Chapter 11 Sections 11.15 to 11.21 of Text 1)	(20 hours)
Module 2:	**Multivariable Differential Calculus** The directional derivative, directional derivatives and continuity, the total derivative, the total derivative expressed in terms of partial derivatives, An application of complex- valued functions, the matrix of a linear function, the Jacobian matrix, the chain rate matrix form of the chain rule. (Chapter 12 Sections. 12.1 to 12.10 of Text 1)	(20 hours)
Module 3:	Implicit functions and extremum problems, the mean value theorem for differentiable functions, a sufficient condition for differentiability, a sufficient condition for equality of mixed partial derivatives, functions with non-zero Jacobian determinant, the inverse function theorem (without proof), the implicit function theorem (without proof), extrema of real-valued functions of one variable, extrema of real- valued functions of several variables. Chapter 12 Sections-. 12.11 to 12.13. of Text 1 Chapter 13 Sections-. 13.1 to 13.6 of Text 1	(25 hours)
Module 4:	Manifolds, fields and forms on manifolds, Stokes' theorem on manifolds, the volume element. Green's theorem(statement only) (Chapter 5-Text II)	(25 hours)

Question Paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	3	1
Total	12	10	4

References:-

1. Limaye Balmohan Vishnu, Multivariate Analysis, Springer.
2. Satish Shirali and Harikrishnan, Multivariable Analysis, Springer.

PG4MATE03

(5 Hrs/week) **DIFFERENTIAL GEOMETRY** **(80marks)**

Text Book: John A. Thorpe, Elementary Topics in Differential Geometry

Module 1:	Graphs and level sets, vector fields, the tangent space, surfaces, vector fields on surfaces, orientation. (Chapters 1 to 5 of the text)	(15 hours)
Module 2:	The Gauss map, geodesics, Parallel transport (Chapters 6, 7 & 8 of the text)	(20 hours)
Module 3:	The Weingarten map, curvature of plane curves, Arc length and line integrals (Chapters 9, 10 & 11 of the text)	(25 hours)
Module 4:	Curvature of surfaces, Parametrized surfaces, local quivalence of surfaces and Parametrized surfaces. (Chapters 12, 14 & 15 of the text)	(30 hours)

Question Paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	3	1
Total	12	10	4

References:-

1. Serge Lang, Differential Manifolds
2. I.M. Siger, J.A Thorpe, Lecture notes on Elementary topology and Geometry, Springer – Verlag, 1967.
3. S. Sternberg, Lectures on Differential Geometry, Prentice-Hall, 1964.
4. M. DoCarmo, Differential Geometry of curves and surfaces.

PG4MATE04

(5 Hrs/week) **THEORY OF WAVELETS** **(80 marks)**

Text Book:- Michael W. Frazier, An introduction to Wavelets through Linear Algebra, Springer- verlag, 2000.

Pre-requisites:- Linear Algebra, Discrete Fourier Transforms, Elementary Hilbert Space theorem.
(No questions shall be asked from these sections.)

Module 1:	Construction of Wavelets on Z_N: The First Stage. (Chapter – 3 Section 3.1 of the text)	(20 hours)
Module 2:	Construction of Wavelets on Z_N : The Iteration Step, Examples – Haar, Shannon and Daubechies). (Chapter – 3 Section 3.2 & 3.3 of the text)	(20 hours)
Module 3:	$l^2(Z)$ Complete Orthonormal sets in Hilbert Spaces, $L^2[-\pi, \pi]$ and Fourier Series. (Chapter – 4 Section 4.1, 4.2 & 4.3 of the text)	(20 hours)
Module 4:	The Fourier Transform and Convolution on $l^2(Z)$ First-stage Wavelets on Z, The Iteration step for Wave lets on Z, Examples-Haar and Daubechies. (Chapter – 4 Section 4.4, 4.5, 4.6 & 4.7 of the text)	(30 hours)

Question Paper Pattern

	Part A	Part B	Part C
.	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	3	1
Total	12	10	4

References:-

1. Mayer, Wavelets and Operators, Cambridge University Press, 1993.
2. Chui, An Introduction to Wavelets, Academic Press, Boston, 1992.

PG4MATE
(80 marks)

(5 Hrs/week)

CLASSICAL MECHANICS

Text: L. D. Landau and E. M. Lifshitz - MECHANICS, (Third Edition)
(Butter worth – Heinenann)

Module 1:	Generalized coordinates, the Principle of least action, Galileo's relativity principle, the Legrangian for a free particle, Legrangian for a system of particle, energy, momentum, centre of mass, angular momentum, motion in one dimension, determination of the potential energy from the period of oscillation, the reduced mass, motion in a central field. (Section 1 to 9, 11 to 14 of the text)	25
Module 2:	Free oscillation in one dimension, angular velocity, the inertia tensor, angular momentum of a rigid body, the equation of motion of a rigid body, Eulerian angle, Euler's equation. (Section 21, 31 to 36 of the text)	20
Module 3:	The Hamilton's equation, the Routhian, Poisson brackets, the action as a function of the coordinates, Maupertui's principle. (Section 40 to 44 of the text)	20
Module 4:	The Canonical transformation, Liouville's theorem, the Hamiltonian – Jacobi equation, separation of the variables, adiabatic invariants, canonical Variables (Section 45 – 50 of the text)	25

Question Paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	2	1
Module IV	3	3	1
Total	12	10	4

References

1. M. G. Calkin, Lagrangian and Hamiltonian Mechanics, Allied
2. Herbert Goldstein, Classical mechanics, Narosa
3. K C Gupta, Classical mechanics of particles and Rigid Bodies, Wiley Eastern

PG4MATE

(5 Hrs/week) **(80 marks)**

PROBABILITY THEORY

All questions shall be based on the relevant portions of the reference books given in the end of each module

Module 1	Discrete Probability (Empirical, Classical and Axiomatic approaches), Independent events, Bayes theorem, Random variables, and distribution functions (univariate and multivariate), Expectation and moments, marginal and conditional distributions. Probability Inequalities (Chebychev, Markov). Modes of convergence, Weak and Strong laws of large numbers (Khintchine's Weak Law , Kolmogrov Strong Law, Bernaulli's Strong Law) Central Limit theorem (Lindeberg-Levy theorem). **References.** 1. S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, 11th Ed., Sultan Chand & Sons, 2011. 2. V.K. Rohatgi, An Introduction to Probability Theory and Mathematical Statistics, 2nd Ed. Wiley Eastern Ltd., 1986.	20
Module 2	Standard discrete and continuous univariate distributions (Binomial, Poisson, Negative binomial, Geometric, Exponential, Hypergeometric, Normal, Rectangular, Cauchy's, Gamma, Beta,), Multivariate normal distribution, Wishart distribution and their properties. **References.** **For univariate distributions, refer the book** S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, 11th Ed., Sultan Chand & Sons, 2011. **For Multivariate distributions, refer the book** T.W. Anderson, An Introduction to Multivariate Statistical Analysis, 3rd Ed., Wiley Interscience, 2003.	20
Module 3	Methods of estimation, properties of estimators, Cramer-Rao inequality, Fisher-Neyman criterion for sufficiency, Rao-Blackwell theorem, completeness ,method of maximum likelihood, properties of maximum likelihood estimators , method of moments. Tests of hypothesis: most powerful and uniformly most powerful tests (Neyman – Pearson Lemma). **References.** **For Estimation, refer the book**: S.C. Gupta and V.K. Kapoor, Fundamentals of Mathematical Statistics, 11th Ed., Sultan Chand & Sons, 2011. **For Tests of Hypothesis, refer the book** : V.K. Rohatgi, An Introduction to Probability Theory and Mathematical Statistics, 2nd Ed. Wiley Eastern Ltd., 1986.	25

Module 4	Gauss-Markov models, estimability of parameters, best linear unbiased estimators, Analysis of variance and covariance. One way and two way classification with one observation per cell. **References.** 1. D.D. Joshi, Linear Estimation and Design of Experiments, Wiley Eastern Ltd., 1990. 2. C.R. Rao, Linear Statistical Inference and its Applications, John Wiely, New York. ,1965. 3. W.G.Cochran and G.M. Cox , Experimental Designs, 2^{nd} Ed., John Wiely, New York. , 1957.	25

Question Paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	2	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	3	1
Total	12	10	4

PG4MATE

(80 marks)

(5 Hrs/week)

SPECIAL FUNCTIONS

Text Book:- Earl. D. Rainville, Special functions, Chelsa Publishing Company, New York, 1960

Module 1	Infinite products:- Introduction, definition of an infinite product, a necessary condition for convergence, the associated series of logarithms, absolute convergence, uniform convergence. The Gamma and Beta functions:- The Euler and Mascheroni constant γ , the Gamma function, a series for ^{1}(z)/(z), evaluation of (1) and ^{1}(1), the Euler product for (z), the difference equation (z + 1) = z (z) the order symbols o and 0, evaluation of certain infinite products, Euler's integral for (z), the Beta function, the value of (z) (1 - z), the factorial function, Legendre's duplication formulae, Gauss' multiplication theorem, a summation formula due to Euler, the behavior of log (z) for large *z* (Chapter 1 & 2 of text – Sections 1 to 22)	25
Module 2	The hypergeometric function:- The function F(a,b,c,z), a simple integral form, F(a,b,c,1) as a function of the parameters, evaluation of F(a,b,c,1), the contiguous function relations, the hypergeometric differential equation, logarithmic solution of the hypergeometric equation, F(a,b,c,z) as a function of its parameters, elementary series multiplications, simple transformations, relation between functions of z and 1 – z. (Chapter 4 of the text – Sections 29 to 39)	20
Module 3	Generalized Hypergeometric Functions: The function ${}_pF_q$, the exponential and binomial functions, a differential equation, other solutions of the differential equation, the contiguous function relations, a simple integral, the ${}_pF_q$ with unit argument. The Confluent Hypergeometric Functions: Basic properties of the ${}_1F_1$, Kummer's first formula, Kummer's second formula. (Chapter 5 – Sections 44 to 50, Chapter 7 - Sections – 68, 69, 70)	25
Module 4	Legendre Polynomials: A generating function, Differential recurrence relations, the pure recurrence relation, Legendre's differential equation, the Rodrigue's formula, Bateman's generating function, additional generating functions, Hypergeometric forms of $p_n(x)$, Brafman's generating function, special properties of $p_n(x)$. Hermite Polynomials: Definition of $H_n(x)$, recurrence relations, the Rodrigue's formula, other generating functions, integrals.	20

Question Paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	2	1
Total	12	10	4

References:-

1. M.A. Pathan, V.B.L.Chaurasia, P.K.Banerji, M.C.Goyal ,Special Functions and Calculus of Variations, Ramesh Book Depot, New Delhi, 2007.
2. Z.X. Wang, D.R. Guo, Special Functions, World Scientific Publishing Company, London, 1989.
3. N.M. Temme, Special Functions – An Introduction to the Classical Functions of Mathematical Physics, John Wiley & Sons, New York, 1996.
4. A.M. Mathai, H.J. Haubold, Special Functions for Applied Scientist, Springer, New York, 2008.

PG4MATE
(80 marks)

(5 Hrs/week)

COMMUTATIVE ALGEBRA

Text Book :- Gregor Kemper, A Course in Commutative Algebra, Springer, ISSN0072-5285, ISBN978-3-642-03544-6

Module 1:	**The Algebra-Geometry Lexicon** – Hilbert's Nullstellensatz Maximal ideals, Jacobson Rings, Coordinate Rings, Simple problems. (Chapter1 Sections 1.1, 1.2 & 1.3 of the text)	(25 hours)
Module 2:	**Noetherian and Artinian Rings.-**The Noether and Artin Properties for Rings and Modules, Notherian Rings and Modules, Simple problems (Chapter 2 Sections 2.1 & 2.2, of the text)	(20 hours)
Module 3:	**The Zariski Topology -** Affine Varieties, Spectra, Noetherian and Irreducible Spaces, Simple problems. (Chapter3 Sections 3.1, 3.2 & 3.3 of the text)	(25 hours)
Module 4:	**A Summary of the Lexicon -** True Geometry: Affine Varieties, Abstract Geometry : Spectra , Simple problems (Chapter4 Sections 4.1 & 4.2, of the text).	(20 hours)

Question Paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	2	1
Total	12	10	4

References: -

1. William W. Adams, Phillippe Loustaunau, An Introduction to Grobner bases, Graduate Studies in Mathematics 3, American Mathematical Society, 1994.

2. Michael F Attiyah, Ian Grant Macdonald, Introduction to Commutative Algebra, Addison- Wesley, Reading, 1969.

3. Nicolas Bourbaki, General Topology, Chapters – 1 – 4, Springer, Berlin, 1993.

PG4MATE

(80 marks)

(5 Hrs/week)

CODING THEORY

Text :- Vera Pless 3rd Edition , Introduction to the theory of error coding codes, Wiley Inter Science

Module 1:	Introduction Basic Definitions Weight, Maximum Likelihood decoding Synarome decoding, Perfect Codes, Hamming codes, Sphere packing bound, more general facts. (chapter 1 & Chapter 2 Sections 2.1, 2.2, 2.3 of the text)	(25 hours)
Module 2:	Self dual codes, The Golay codes, A double error correction BCH code and a field of 16 elements. (Chapter 2 Section 2.4 & Chapter 3 of the text)	(20 hours)
Module 3:	Finite fields (Chapter 4 of the text)	(20 hours)
Module 4:	Cyclic Codes, BCH codes (Chapter 5 & Chapter 7 of the text)	(25 hours)

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	2	1
Module IV	3	3	1
Total	12	10	4

References:-

1. R-Lidi, G. Pliz, Applied Abstract Algebra, Springer Verlag.
2. J.H.Van Lint, Introduction to Coding Theory, Springer Verlag
3. R.E.Blahut, Error- Control Codes, Addison Wesley.

PG4MATE
(80marks)

(5 Hrs/week)

ALGEBRAIC GEOMETRY

Text:- Brendan Hassett, Intoduction to Algebraic Geometry, Cambridge University Press, 2007.

Module 1:	**Guiding problems:** Implicitization, Ideal membership, Interpolation**.** **Division algorithm and Grobner bases** 3and chain 47onditions, Buchberger's Criterion. (Chapter 1 – Sections 1.1 to 1.3, Chapter – 2 Sections 2.1 to 2.5)	(30 hours)
Module 2:	**Affine varieties:** Ideals and varieties, Closed sets and the Zariski topology, Coordinate rings and morphisms, Rational maps, Resolving rational maps, Rational and unirational varities. (Chapter – 3 Sections 3.1 to 3.6)	(22 hours)
Module 3:	**Elimination:** Projections and graphs, Images of rational maps, Secant varieties, joins, and scrolls. **Resultants:** Common roots of univariate polynomials, The resultant as a function of the roots, Resultants and elimination theory.(Chapter – 4 Sections 4.1 to 4.3 Chapter – 5 Sections 5.1 to 5.3)	(23 hours)
Module 4:	**Irreducible varieties**: Existence of the decomposition, Irreducibility and domains, Doeminant morphisms. **Nullstellensatz:** Statement of the Nullstellensatz, Classification of maximal ideals, Transcendence bases, Integral elements. **Primary decomposition:** Irreducible ideals, Quotient ideals, Primary ideals.(Chapter:- 6 Sections 6.1 to 6.3, Chapter – 7 Sections 7.1 to 7.4, Chapter – 8 Sections 8.1 to 8.3)	(15 hours)

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	2	1
Total	12	10	4

Reference:

1. William Fulton, Algebraic Curves: An Introduction to Algebraic Geometry, Advanced Book Program, Redwood City, CA: Addison-Wesley, 1989.
2. Phillip Griffiths and Joseph Harris, Principles of Algebraic Geometry, New York: Wiley-Interscience, 1978.
3. Joe Harris, Algebraic Geometry, Graduate Texts in Mathematics, 133. New York: Springer-Verlag, 1992

FRACTAL GEOMETRY PG4MATE

(80 marks)

(5 Hrs/week)

Text:- Kenneth Falconer, FRACTAL GEOMETRY Mathematical Foundations and Applications, John Wiley & Sons, New York.

Pre-requisites – Mathematical background – A quick revision (Chapter 1 of the text). No questions shall be asked from this section. (5 hours)

Module 1:	**Hausdorff measure and dimension:** Hausdorff measure, Hausdorff dimension, Calculation of Hausdorff dimension-Simple examples, Equivalent definitions of Hausdorff dimension, Finer definitions of dimension. **Alternative definitions of dimension:** Box counting dimension, Properties and problems of box counting dimension, Modified box counting dimension, Packing measures and dimension. (Chapter 2 , 3 Sections 3.1 to 3.4 of the text.)	(30 hours)
Module 2:	**Techniques for calculating dimensions:** Basic methods, Subsets of finite measure, Potential theoretic methods, Fourier transform methods. **Local structure of fractals:** Densities, Structure of 1-sets, Tangents to s-sets. (Chapter 4 & 5 of the text.)	(25 hours)
Module 3:	**Projections of fractals:** Projections of arbitrary sets, Projections of s-sets of integral dimension, **Products of fractals** : Product formulae (Chapter 6 & 7 of the text)	(20 hours)
Module 4:	**Intersections of fractals:** Intersection formulae for fractals, Sets with large intersection. (chapter 8 of the text)	(15 hours)

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	3	1
Module III	3	2	1
Module IV	3	2	1
Total	12	10	4

Reference:-

1. Falconer K.J, The Geometry of Fractal sets, Cambridge University Press, Cambridge.
2. Barnsley M.F, (1988), Fractals every where, Academic press, Orlando, FL.
3. Mandelbrot B.B, (1982), The Fractal Geometry of Nature, Freeman, San Francisco.
4. Peitgen H.O and Richter P.H, (1986), The Beauty of Fractals, Springer, Berlin.

PG4MATE
(80 marks)
(5 Hrs/week)

LIE ALGEBRAS

Text:- James E. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer

Module 1:	**Basic Concepts:** Definition and first examples, Ideals and homomorphisms, Solvable and nilpotent Lie Algebras. (Chapter I Sections 1, 2, & 3 of the text)	(25 hours)
Module 2:	**Semi simple Lie Algebras:** Theorems of Lie and Cartan, Killing form, Complete reducibility of representations. (Chapter II Sections 4, 5, & 6 of the text)	(20 hours)
Module 3:	**Root Systems:** Axiomatics, Simple roots and Weyl group, Classification. (proof of Classification theorem excluded) (Chapter III Sections 9, 10 & 11 of the text)	(25 hours)
Module 4:	**Isomorphism and Conjugacy Theorems:** Isomorphism theorem, Cartan Algebras, Conjugacy theorems (Chapter IV Sections 14, 15, & 16 – 16.1 to 16.3 of the text)	(20 hours)

Question paper Pattern

	Part A	Part B	Part C
	Short questions	Short essays	Long essays
Module I	3	3	1
Module II	3	2	1
Module III	3	3	1
Module IV	3	2	1
Total	12	10	4

References:-

1. J.G.F. Belinfante and B. Kolman, Asurvey of Lie Groups and Lie Algebras with computational methods and Applications, .Philadelphia : SIAM, 1972.
2. N. Jacobson, Lie Algebras, New York – London, Wiley interscience, 1962.
3. H. Samuelson, Notes on Lie Algebras, Van Nostrand Reinhold Mathematical studies No. 23, New York: Van Nostrand Reinhold, 19